FINAL EXAM (RIBET)

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(1) **TRUE/FALSE**

(a) If A is a square invertible matrix, then A and A^{-1} have the same rank

(b) If A is an $m \times n$ matrix and if b is in $\mathbf{R}^m$, there is a unique $x \in \mathbb{R}^n$ for which $\|A\mathbf{x} - \mathbf{b}\|$ is smallest.

(c) If A is an $n \times n$ matrix, and if $\mathbf{v}$ and $\mathbf{w}$ satisfy $A\mathbf{v} = 2\mathbf{v}$, $A\mathbf{w} = 3\mathbf{w}$, the $\mathbf{v} \times \mathbf{w} = 0$

(d) If the dimensions of the null spaces of a matrix and its transpose are equal, then the matrix is qsquare

(e) If A is a 2×2 matrix, then -1 cannot be an eigenvalue of A^2.

(f) I likes the linear algebra portion of this course more than the differential equations portion

(g) If 4 linearly independent vectors lie in $Span\{\mathbf{w_1}, \cdots, \mathbf{w_n}\}$, then n must be at least 4.

(h) If B is invertible, then the column spaces of A and AB are equal.

(i) If A is a matrix, then the row spaces of A and $A^T A$ are equal

(j) If 2 symmetric $n \times n$ matrices A and B have the same eigenvalues, then $A = B$

(k) If the characteristic polynomial of A is $p(\lambda) = (\lambda - 1)(\lambda + 1)(\lambda - 3)^2$, then A *has* to be diagonalizable

Date: Wednesday, December 7th, 2011.

(2) Consider the following vectors:

$$\mathbf{v_1} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Find $\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}$ such that $\{\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}\}$ is an orthogonal basis for $Span\,\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$.

(3) Solve the following system of differential equations:

$$\begin{cases} x_1'(t) = -2x_1(t) + 2x_2(t) \\ x_2'(t) = 2x_1(t) + x_2(t) \end{cases}$$

and $x_1(0) = -1, x_2(0) = 3$.

(4) Find bases for $Nul(A), Row(A), Col(A)$, where:

$$A = \begin{bmatrix} 1 & 1 & 3 & 2 \\ 3 & 1 & 1 & 0 \\ 4 & 2 & 4 & 2 \end{bmatrix}$$

(5) Find the first 4 terms A_0, A_1, A_2, A_3 of the Fourier cosine series of $f(x) = |\sin(x)|$

Hint: $\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

(6) Solve the following PDE:

$$\begin{cases} \dfrac{\partial u}{\partial t} = 25\dfrac{\partial^2 u}{\partial x^2} & 0 < x < \pi, \quad t > 0 \\ u(0,t) = u(\pi,t) = 0 & t > 0 \\ u(x,0) = \sin(3x) - \sin(4x) & 0 < x < \pi \end{cases}$$

(7) Suppose $\mathbf{v_1}, \cdots, \mathbf{v_n}$ are vectors in $\mathbb{R}^n$ and that A is an $n \times n$ matrix.
If $A\mathbf{v_1}, \cdots, A\mathbf{v_n}$ form a basis for $\mathbb{R}^n$, show that $\mathbf{v_1}, \cdots \mathbf{v_n}$ form a basis of $\mathbb{R}^n$ and that A is invertible.

(8) Let $\mathbf{v_1} = \begin{bmatrix} 0 \\ 5 \\ -2 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v_3} = \begin{bmatrix} 9 \\ 8 \\ 7 \end{bmatrix}$

Suppose A is the 3×3 matrix for which $A\mathbf{v_1} = \mathbf{v_1}$, $A\mathbf{v_2} = \mathbf{0}$, $A\mathbf{v_3} = 5\mathbf{v_3}$. Find an invertible matrix P and a diagonalizable matrix D such that $A = PDP^{-1}$.